

Conclusions. Because of the original properties TMW found widespread use and market dynamic of consumption (demand exceeds supply in order [1]). The technologies, usually created by the method of "trial and error" and not perfect. To develop effective modes of production TTW necessary to explore the distribution of moisture and temperature fields and heat treatment process to intensify the search for effective ways to prevent avoid possible ignition of wood that is exposed to thermal modification.

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Термічно модифікована деревина: технології, тенденції та перспективи розвитку

Розглянуто історію тенденції та перспективи розвитку термічного модифікування деревини, фізико-механічні та експлуатаційні властивості продукту, короткий огляд технологій. Проведено аналіз динаміки світового ринку споживання термічно обробленої деревини; розглянуті можливі напрямки інтенсифікації процесу розробки ефективних режимів її виготовлення.

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MATHEMATICAL SIMULATION OF SURFACE SHAPE FOR REAL LOG

The features of the mathematical simulation of surface shape for real log were analyzed on the basis of surface shape scanning for log cross sections along its length. The possible shift of the log axis under the scanning relative to its geometric (regressive) axis was taken into account in developed mathematical model.

Key words: log, cross section, scanning, surface shape (form), mathematical simulation, trigonometric polynomial, geometric centre of mass, geometric (regressive) log axis.

Statement of problems and research currency. The significant research work for search the optimal ways of rational logs sawing into sawn timber was performed. Effective solution of this problem requires not only the available the reliable information about size and quality characteristics of logs and sawn timber but also its correct interpretation. The decision concerning direction choice, method and sawing scheme of logs into sawn timber based on information about their size and quality characteristics. This decision is should be also sustained by mathematical and technological calculations. Therefore, information about dimensional characteristics of real logs (mathematical simulation of surface shape of

real logs, in particular) as well as availability of quality characteristics used for decision concerning sawing variants (direction, method and scheme) can be served as a main pre-condition of rational log sawing into sawn timber. Taking to account all abovementioned factors this direction of researches is very actually.

Analysis of the available scientific data. Two directions of mathematical description of logs for further development of their mathematical models used for prediction of sawn-timber recovery are mainly considered by researchers. The first direction involves the use of complex mathematical methods but it provides an exact or approximate to it mathematical description of surface shape of real logs [1-3]. However, most researches [4-9] taking to account the complexity of first direction of investigations use the second direction. The log is considered as a geometric figure of revolution (circular or elliptic truncated paraboloid of revolution, truncated cone of revolution, channel surface with variable generatrix or figure generated by generatrix revolution) in this direction.

It is much easier to study as compared with first but the use of this direction is limited by different factors. Particularly it is not taking to account the features of surface shape of real logs such as fluctuation of dimensional log characteristics caused by natural conditions of tree growth. That's why it can be used only for preliminary prediction of sawn-timber recovery. The reliability of the mathematical description of surface shape for real logs depends on correct interpretation of available data about dimensional characteristics of logs whereas results of investigations [1-9] indicate that at this time not developed appropriate method for creation of effective mathematical models of real logs.

Note that the process of rational log sawing into sawn-timber must be involve not only reliable mathematical description of surface shape for real logs but it also includes the consideration of technological features of this process.

Method of mathematical simulation of surface shape for real log. Results of surface shape scanning for log cross sections along its length (fig. 1) can be presented as follows:

$$\left\{ x_j^{(i)}, y_j^{(i)} \right\} \Big|_{j=1, \overline{m}, i=0, \overline{N}}, \quad (1)$$

where: $x_j^{(i)}, y_j^{(i)}$ – the coordinates of j^{th} points of scanning on the i^{th} log cross section along its length; m – number of scanning points on each log cross section along log length; N – number of plane cross sections of log along its length ($z=0, z=h, \dots, z=Nh$, where h – scanning step; plane $z=0$ passes through the top diameter of log and plane $z=Nh$ passes through bottom diameter of log). Enter the term “conditional center of log cross section” $C^{(i)}(cx^{(i)}, cy^{(i)}, ih)$ and consider the surface shape of log cross sections, each of which describes by eight scanning points ($m=8$).

Assuming that:
$$cx^{(i)} = \frac{1}{8} \sum x_j^{(i)}, \quad cy^{(i)} = \frac{1}{8} \sum y_j^{(i)}. \quad (2)$$

False co-ordinates of scanning points after displacement to conditional center of log cross section $C^{(i)}$ are determined as follows:

$$\begin{cases} u_j^{(i)} = x_j^{(i)} - cx^{(i)} \\ v_j^{(i)} = y_j^{(i)} - cy^{(i)} \end{cases} \quad (3)$$

Rotational positions of scanning points ($r_j^{(i)}, \phi_j^{(i)}$) are defined based on false co-ordinates of scanning points after their displacement to conditional center of log cross

section $C^{(i)}$:

$$r_j^{(i)} = \sqrt{u_j^{(i)^2} + v_j^{(i)^2}}, \quad \varphi_j^{(i)} = \arctg \left(\frac{v_j^{(i)}}{u_j^{(i)}} \right) + \Delta\varphi_j^{(i)}, \quad (4),$$

where: $\Delta\varphi_j^{(i)} = -\pi$, if $u_j^{(i)} < 0$, $v_j^{(i)} < 0$; or $\Delta\varphi_j^{(i)} = \pi$, if $u_j^{(i)} < 0$, $v_j^{(i)} > 0$ and equals zero in other cases.

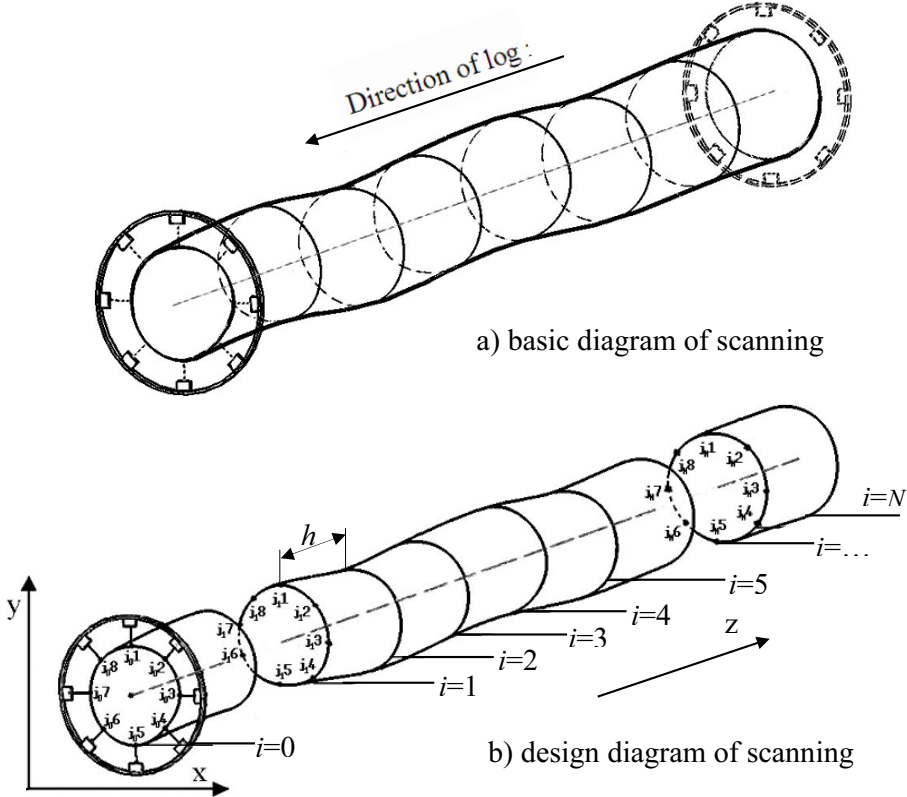


Fig. 1. Scanning of surface shape of log cross sections along its length

Scanning points is described as a partial sum of Fourier trigonometric polynomial which approximates the contour line of form for i^{th} log cross section (fig. 2):

$$r^{(i)} = f^{(i)}(\varphi) = A_0^{(i)} + \sum_{k=1}^3 A_k^{(i)} \cos k\varphi + \sum_{k=1}^4 B_k^{(i)} \sin k\varphi, \quad (5)$$

where: $A_0^{(i)}, A_k^{(i)}, B_k^{(i)}$ – unknown coefficients of a trigonometric polynomial.

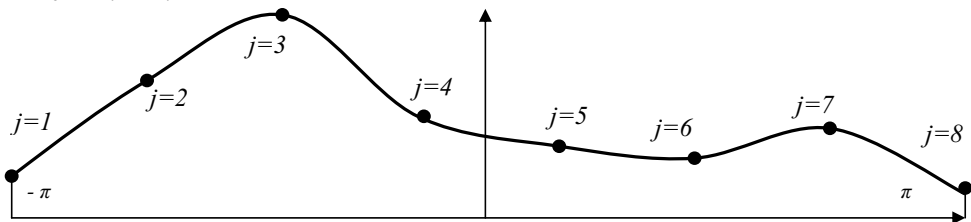


Fig. 2. Design diagram of contour line approximation of form for i^{th} log cross section by trigonometric polynomial

Substituting in (5) the co-ordinates of eight scanning points for each cross section, we obtain the system of equations:

$$A_0^{(i)} + \sum_{k=1}^3 A_k^{(i)} \cos k\varphi_j^{(i)} + \sum_{k=1}^4 B_k^{(i)} \sin k\varphi_j^{(i)} = r_j^{(i)}, \quad j = \overline{1,8}. \quad (6)$$

Solving a system of eight equations with eight unknown quantities, we find the coefficients of trigonometric polynomial (5) which describes the contour of form for i^{th}

$$\text{log cross section based on scanning points:} \quad \begin{cases} A_k^{(i)} & k = \overline{0,3}, \\ B_k^{(i)} & k = \overline{1,4}. \end{cases} \quad (7)$$

The area of i^{th} log cross section for the scanning points is determined from the

$$\text{formula:} \quad S^{(i)} = \frac{1}{2} \int_{-\pi}^{+\pi} r^2(\varphi) d\varphi. \quad (8)$$

Due to orthogonality of trigonometric system $\{1, \cos k\varphi, \sin k\varphi\}$ on $[-\pi, \pi]$ the quantity (8) is easily calculated as:

$$S^{(i)} = \frac{\pi}{2} \left(2A_0^{(i)2} + A_1^{(i)2} + A_2^{(i)2} + A_3^{(i)2} + B_1^{(i)2} + B_2^{(i)2} + B_3^{(i)2} + B_4^{(i)2} \right). \quad (9)$$

Co-ordinates of the geometric center of the mass for i^{th} log cross section is determined according to formulas:

$$u_c^{(i)} = \frac{1}{2S} \int_{-\pi}^{+\pi} r^3(\varphi) \cos \varphi d\varphi, \quad v_c^{(i)} = \frac{1}{2S} \int_{-\pi}^{+\pi} r^3(\varphi) \sin \varphi d\varphi. \quad (10)$$

These formulas can be calculated by direct integration or they can be found by trapezoid rule using within selected mathematical package. The volume of routine work is considerably decreased as a result of second operation application.

Turning to the initial reference system we obtained the co-ordinates of the geometric center of the mass for i^{th} log cross section (fig. 3):

$$x_c^{(i)} = cx^{(i)} + u_c^{(i)}, \quad y_c^{(i)} = cy^{(i)} + v_c^{(i)}. \quad (11)$$

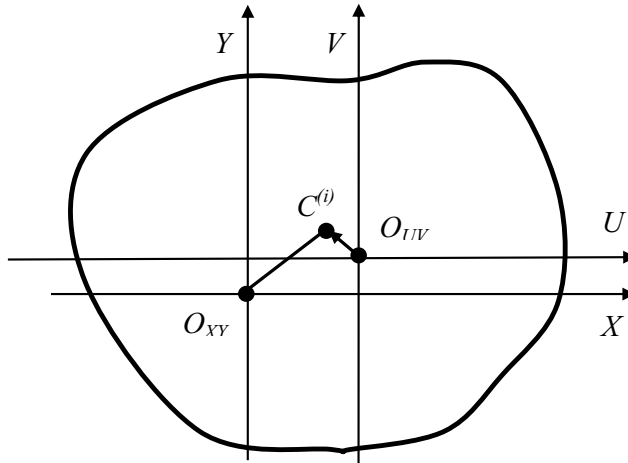


Fig. 3. Scheme of co-ordinates recalculation of the geometric center of the mass for i^{th} log cross section concerning initial reference system

After determination the co-ordinates of the geometric center of mass, we introduce the relative co-ordinates of scanning points relative to the geometric center of mass

for each section:

$$\begin{cases} u_j^{(i)} = x_j^{(i)} - x_c^{(i)} \\ v_j^{(i)} = y_j^{(i)} - y_c^{(i)} \end{cases}$$

Using formulas (4–11) for the repeated recalculation, we get the co-ordinates of corrected center of mass for i^{th} log cross section $\mathbf{C}^{(i)}(cx^{(i)}, cy^{(i)}, ih) \in \mathbf{R}^3$, the area of i^{th} log cross section $\mathbf{S}^{(i)}$ and coefficients of trigonometric polynomial (5) which describes the contour of form for i^{th} log cross section for scanning points (function $\mathbf{r}^{(i)} = \mathbf{f}^{(i)}(\varphi)$, presented by four coefficients $A_k^{(i)} (k = \overline{0,3})$ and four coefficients $B_k^{(i)} (k = \overline{1,4})$).

Determination of the geometric axis of log. The geometric axis of real log in most cases has a curvature which caused by natural fluctuations of dimensional characteristics of log and its use is non-effective for further study of log sawing. So we propose that the geometric axis of real log can be presented by linear regression equation which provides appropriate description of geometric centers of mass of each i^{th} log cross section along its length and will be effective for further researches of log sawing. The term “regressive axis of log” can be applied for definition of geometric axis of log which presented by linear regression equation.

For determination of linear regression equation which describes the geometric centers of mass of each i^{th} log cross section along its length the co-ordinates of the geometric centers of mass of each i^{th} log cross section along its length are renamed as $(u_i = x_c^{(i)}, v_i = y_c^{(i)})$. Then a linear regression equation can be represented in parametric form of regression system of abscissa ($u = ai + b$) and ordinate ($v = ci + d$) on applicate z ($z = ih$). Coefficients a, b, c, d of regression equations are determined by least square method [10]:

$$\sum_{i=0}^N (u_i - ai - b)^2 \rightarrow \min; \quad \sum_{i=0}^N (v_i - ci - d)^2 \rightarrow \min; \quad (12)$$

$$\begin{cases} a \sum_{i=0}^N i^2 + b \sum_{i=0}^N i = \sum_{i=0}^N i u_i \\ a \sum_{i=0}^N i + b(N+1) = \sum_{i=0}^N u_i \end{cases}; \quad \begin{cases} c \sum_{i=0}^N i^2 + d \sum_{i=0}^N i = \sum_{i=0}^N i v_i \\ c \sum_{i=0}^N i + d(N+1) = \sum_{i=0}^N v_i \end{cases}, \quad (3)$$

where $\sum_{i=0}^N i = 0 + \dots + N = \frac{N(N+1)}{2}$, $\sum_{i=0}^N i^2 = 0^2 + \dots + N^2 = \frac{N(N+1)(2N+1)}{6}$.

Solving (13) by Cramer formulas, we find:

$$\Delta = \left(\sum_{i=0}^N i^2 \right) (N+1) - \left(\sum_{i=0}^N i \right)^2 = \frac{N(N+1)^2(2N+1)}{6} - \frac{N^2(N+1)^2}{4} = \frac{(N+1)^2}{12} N(N+2)$$

Thus:

$$\begin{cases} a = \frac{1}{\Delta} \left((N+1) \sum_{i=0}^N i u_i - \frac{N(N+1)}{2} \sum_{i=0}^N u_i \right); \\ b = \frac{1}{\Delta} \left(\frac{N(N+1)(2N+1)}{6} \sum_{i=0}^N u_i - \frac{N(N+1)}{2} \sum_{i=0}^N i u_i \right); \end{cases} \quad (14)$$

$$\begin{cases} c = \frac{1}{\Delta} \left((N+1) \sum_{i=0}^N iv_i - \frac{N(N+1)}{2} \sum_{i=0}^N v_i \right); \\ d = \frac{1}{\Delta} \left(\frac{N(N+1)(2N+1)}{6} \sum_{i=0}^N v_i - \frac{N(N+1)}{2} \sum_{i=0}^N iv_i \right). \end{cases} \quad (15)$$

Let directing vector of regressive axis of log through $\vec{\ell}(a, c, h)$.

Deviation of the log (regressive) axis concerning the scanning axis (fig. 4) can be

determined from the dependence: $tg\alpha = \frac{\sqrt{a^2 + c^2}}{h}, \quad (16)$

where: α – angle of deviation of the log axis concerning the scanning axis; a , c – angular coefficients of deviation at the X-axis and Y-axis respectively.

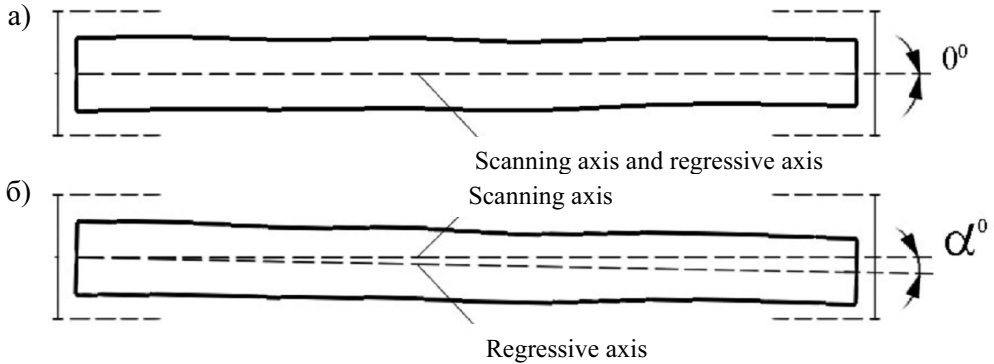


Fig. 4. Scheme of log scanning: a) the log (regressive) axis and the scanning axis is coincided; b) the log (regressive) axis and the scanning axis is deviated

Quantity $|tg\alpha|$ can be considered as an equality criterion of log location concerning its regressive axis. For particular cases:

- if the log axis is shifted from the scanning axis in the horizontal plane (fig. 5a), that the log must be rotate/shift at the angle $arctg(a/h)$;
- if the log axis is shifted from the scanning axis in the vertical plane (fig. 5b), that the log must be rotate/shift at the angle $arctg(c/h)$.

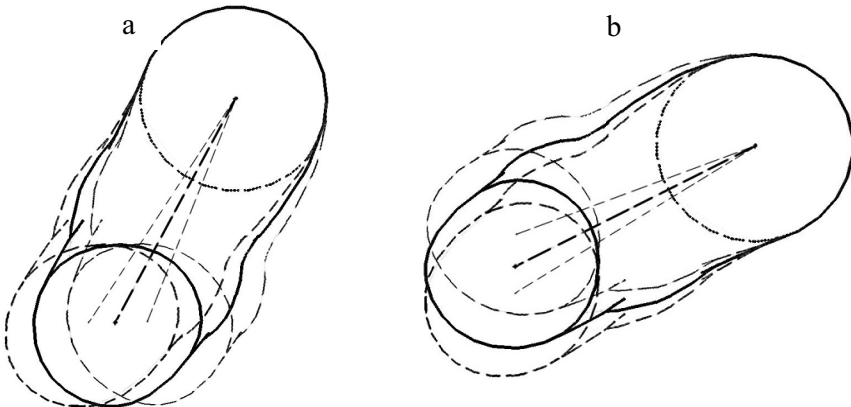


Fig. 5. Scheme of log rotation on the calculated angle:
a) in the horizontal plane; b) in the vertical plane

If the plane of log location is parallel to the cut plane under log sawing then $c>0$ (log sawing is begun from top) and $c<0$ (log sawing is begun from bottom). Instead of, if the plane of log location is perpendicular to the cut plane under log sawing then $a>0$ (log sawing is begun from top) and $a<0$ (log sawing is begun from bottom). These results indicate that the log because of its shape along length (taper) and Earth gravity is mainly based on its generatrix. The forced shifting of certain log part or availability of shape defects of log, strongly pronounced curvature, in particular cannot be considered.

Points of i^{th} cross sections lying on a regressive axis of log have the co-ordinates $(ai+b, ci+d)$. Relative coordinates of the geometric centers of i^{th} cross sections of log concerning to its regressive axis can be represented by such dependencies:

$$\begin{cases} x_{Ci} = u_i - ai - b \\ y_{Ci} = v_i - ci - d \end{cases} \quad (17)$$

Co-ordinates of j^{th} points of log cross section concerning to its regressive log

axis are presented as follows:

$$\begin{cases} u_j^{(i)} = x_j^{(i)} - ai - b \\ v_j^{(i)} = y_j^{(i)} - ci - d \end{cases} \quad (18)$$

In the case of shifting of the log axis relative to its scanning axis the received values of cross section areas are overestimated that's why we calculated corrected co-ordinates of j^{th} points for i^{th} log cross section concerning its linear regressive axis ($\bar{\ell}$).

They can be presented as:

$$\begin{cases} u_j'^{(i)} = u_j^{(i)} \frac{h}{\sqrt{a^2 + h^2}} \\ v_j'^{(i)} = v_j^{(i)} \frac{h}{\sqrt{c^2 + h^2}} \end{cases} \quad (19)$$

The recalculation with using of formulas (4–11) is carried out after determination of corrected co-ordinates of j^{th} points of i^{th} log cross section concerning linear regressive axis of log ($\bar{\ell}$) with consideration of log location. The co-ordinates of corrected center i^{th} log cross section $C_{\bar{\ell}}^{(i)}(cx^{(i)}, cy^{(i)}, ih)$, the area of i^{th} log cross section $S_{\bar{\ell}}^{(i)}$ and the coefficients of trigonometric polynomial, which describes form contour of i^{th} log cross section according to scanning points (function $r^{(i)} = f^{(i)}(\varphi)$ entered through four coefficients A_k ($k = \overline{0,3}$) and four coefficients B_k ($k = \overline{1,4}$) are the results of performed recalculation. In addition, it allows to complete the building of model for real log oriented concerning its geometric (regressive) axis with consideration of cut plane.

Thus, as results of empirical data processing, we get:

1) the co-ordinates of j^{th} scanning points of each i^{th} log cross section concerning its linear regressive axis of log ($\bar{\ell}$) $\left\{ u_j^{(i)}, v_j^{(i)}, ih \right\} \Big|_{j=\overline{1,m}, i=\overline{0,N}}$;

2) the co-ordinates of geometric centers of mass of each i^{th} log cross section concerning linear regression axis of log ($\bar{\ell}$) which can be represented as follows

$$C^{(i)}\{x_{Ci}, y_{Ci}, ih\} \Big|_{i=0, \overline{N}};$$

3) the areas of t^{th} log cross sections with error that occurred due to the difference between linear regressive axis ($\bar{\ell}$) and scanning axis $\{S^{(i)}\} \Big|_{i=0, \overline{N}}$.

Discussion. Use of mathematical simulation of surface shape for real log based on results of scanning of shape for surface of cross sections along log length provides development of a reliable mathematical 3D model of log surface. This model is taken to account the probability of the log axis deviation concerning the scanning axis. The developed mathematical model is effective for prediction of sawn-timber volume recovery and optimization of sawing scheme. In addition, this model is considered natural fluctuations of dimensional characteristics for real log in contrast to mathematical 3D model of surface for real log described by figures of revolution.

Conclusions

1. The mathematical 3D model of surface for real log based on results of scanning of form of cross sections of log along its length is created. Its realization will allow to take into account natural fluctuations of dimensional characteristics for real log. This method is mathematically proved and suitable for further scientific researches, effective prediction of sawn-timber volume recovery and optimization of log sawing scheme in particular.

2. The algorithm of method modification for development of mathematical 3D model of surface for real log which considered the probable deviation of the log axis concerning the scanning axis is proposed.

3. Results of sawing simulation performed with using of mathematical 3D model of surface for log real will allow to define the real influence of different factors (dimensional and qualitative characteristics as well as shape of log; directions, methods and schemes of log sawing; log positioning etc) on the sawn-timber recovery.

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Математичне моделювання форми поверхні реальної колоди

Ефективне вирішення проблеми раціонального розпилювання лісоматеріалів (колод) на пилопродукцію потребує не лише наявності достовірної інформації про їх розмірно-якісні характеристики, але й правильної інтерпретації цієї інформації, зокрема шляхом створення адекватних математичних моделей реальних колод. Незважаючи на проведену значну дослідницьку роботу щодо вирішення даної проблеми, відомі результати досліджень свідчать, що на даний час не розроблено дієвої методики для створення адекватних математичних моделей реальних колод. Прийняття рішення щодо вибору варіанту (напрям, методу, способу і схеми) розпилювання колод на пилопродукцію на підставі інформації про їх розмірно-якісні характеристики повинно бути математично і технологічно обґрунтовано. Тому урахування інформації про розмірні характеристики реальних колод, зокрема математичне моделювання форми (математичний опис) їх поверхонь, нарівні з урахуванням інформації про якісні характеристики, на підставі яких приймається рішення щодо варіанту розпилювання, є основною передумовою раціонального розпилювання колод на пилопродукцію, а сам напрям досліджень – безперечно актуальний.

Розроблено методику математичного моделювання форми поверхні реальної колоди за результатами сканування форми поверхонь поперечних перетинів колоди за її довжиною з урахуванням можливого зміщення осі колоди під час сканування відносно її геометричної (регресійної) осі. Створена за наведеною у роботі методикою математична 3D модель поверхні реальної колоди ефективна для встановлення впливу різних факторів (розмірно-якісних характеристик і форми колод, напрямів, способів та схем розпилювання колод, базування колод тощо) на вихід пилопродукції, прогнозування об'ємного виходу пилопродукції та оптимізації плану розпилювання, і на відміну від математичної 3D моделі поверхні реальної колоди, описаної фігурами обертання, враховує природні флуктуації розмірних характеристик реальної колоди.

Ключові слова: колода, поперечний перетин, сканування, форма поверхні, математичне моделювання, тригонометричний многочлен, геометричний центр мас, геометрична (регресійна) вісь колоди.